DRAFT

Unit of Study
Exponents

Grade: 8	Topic: Exponent operations and rules	Length of Unit: 12 – 15 days

Focus of Learning

Common Core Standards:

Work with Radicals and Integer Exponents

8.EE.1 Know and apply the properties of integer exponents to general equivalent numerical expressions.

8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.

Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understanding(s): *Students will understand that…*

1) Number sense reasoning can generate rules for multiplying and dividing powers with the same base.
2) The rules for multiplying and dividing powers with the same base can generate the meaning of and rules for zero and negative exponents.
3) Operations with scientific notation can be used to solve real world problems.

Guiding Questions: *These questions will guide student inquiry.*

1) Why is using exponents helpful?
2) When is it appropriate to express numbers in scientific notation?
3) How does understanding the exponent rules help you solve real-world problems involving scientific notation?
4) How can we estimate quantities of variable lengths using exponents?

Student Performance

Knowledge: *Students will understand/know…*

- The exponent in an exponential term expresses how many times the base is to be multiplied
- The rules for multiplying and dividing powers with the same base always work
- The proof of $x^0=1$
- Negative exponents can be written as positive exponents using the rules for multiplying and dividing exponents with the same base.
- Scientific notation is used to represent large or small numbers

Application: *Students will be able to…*

- Expand, simplify, and evaluate expressions involving exponents, including products and quotients raised to powers
- Prove the rules of exponents for multiplying and dividing exponents with the same base by using the definition of an exponent.
- Generate and use the rules for multiplying and dividing powers with the same base
- Generate and use the rules for zero exponents and negative exponents
- Express large and small numbers in scientific notation
- Use all exponent rules to perform operations of numbers written in scientific notation

Assessments (attached)
<u>Pre-Assessment:</u> **<u>Formative Interim Assessment:</u>** • Illustrative Mathematics: 8.EE "Extending the Definitions of Exponents, Variation 1" (Use after lesson 5) **<u>Suggested Formative Assessments:</u>** o Illustrative Mathematics: 8.EE "Giantburgers" (Use after lesson 7) o Illustrative Mathematics: 8.EE "Ants versus Humans" (Use after lesson 7) o Smarter Balanced Sample Item: MAT.08.CR.000EE.B.494.C1.TB (Use after lesson 7) **<u>Post Assessment (Culminating Task):</u>** • Exponents "Blood in the Human Body"

Learning Experiences (Lesson Plans Attached)		
<u>Days</u>	**<u>Lesson Sequence</u>**	**<u>Materials</u>**
	Lesson 1: Definition of an Exponent *Students will know:* • The exponent in an exponential term tells us how many times the base is to be multiplied *Students will be able to:* • Expand, simplify, and evaluate expressions involving exponents.	
	Lesson 2: Definition of an Exponent (Continued) *Students will know:* • The exponent in an exponential term tells us how many times the base is to be multiplied *Students will be able to:* • Expand, simplify, and evaluate expressions involving exponents, including products and quotients raised to powers.	
	Lesson 3: Properties for Multiplying and Dividing Exponents with the Same Base *Students will know:* • The rules for multiplying and dividing exponents with the same base always work *Students will be able to:* • Prove the rules of exponents for multiplying and dividing exponents with the same base by using the definition of an exponent. a) $a^m \cdot a^n = a^{m+n}$ b) $\frac{a^m}{a^n} = a^{m-n}$	
	Lesson 4: Properties for Zero and Negative Integer Exponents *Students will know:* • The proof of $x^0=1$ using the properties for multiplying and dividing exponents with the same base (*see lesson 3*). • Negative exponents can be written as positive exponents using the rules for multiplying and dividing exponents with the same base. *Students will be able to:* • Use the rules that they generated in Lesson 3 (for multiplying and dividing exponents with the same base) to generate properties of zero and negative exponents. a) $a^0 = 1$ b) $a^{-m} = \frac{1}{a^m}$	

	Lesson 5: Properties - Review and Assessment *Students will:* • Propose, justify and communicate solutions	**Interim Assessment:** • Illustrative Mathematics: "Extending the Definitions of Exponents, Variation 1"
	Lesson 6: Expressing Number in Scientific Notation *Students will know:* • Scientific notation is used to represent large or small numbers *Students will be able to:* • Express large and small numbers in scientific notation	
	Lesson 7: Using Scientific Notation to Solve Real-World Problems *Students will know:* • Scientific notation is used to represent large or small numbers *Students will be able to:* • Perform operations with numbers expressed in scientific notation, and choose units of appropriate size to represent given measurements. • Use operations with scientific notation that can be used to solve real world problems.	**Suggested Formative Assessments:** • Illustrative Mathematics: 8.EE "Giantburgers" • Illustrative Mathematics: 8.EE "Ants versus Humans" • Smarter Balanced Sample Item: MAT.08.CR.000EE.B.494.C1.TB
	Lesson 8: Review *Students will:* • Propose, justify and communicate solutions	
	Lesson 9: Culminating Task *Students will:* • Show their knowledge and understanding of exponents.	**Post Assessment** • Exponents "Blood in the Human Body"

Resources

Online	Text
Georgia Department of Education https://www.georgiastandards.org/Common-Core/Pages/Math.aspx **Illustrative Mathematics** http://www.illustrativemathematics.org/ **Inside Mathematics/MARS tasks** http://www.insidemathematics.org/ ; http://map.mathshell.org/materials/index.php **National Library of Virtual Manipulatives** http://nlvm.usu.edu/en/nav/vlibrary.html **North Carolina Department of Public Instruction** http://www.dpi.state.nc.us/acre/standards/common-core-tools/#unmath **Progressions for the Common Core State Standards in Mathematics** http://ime.math.arizona.edu/progressions/ **Smarter Balanced Assessment Consortium** http://www.smarterbalanced.org/smarter-balanced-assessments/#item **Utah State Office of Education** http://www.schools.utah.gov/CURR/mathsec/Core/8th-Grade-Core/8-EE-1.aspx	**Prentice Hall Mathematics. *California Algebra.* Boston: Pearson Education, Inc. 2009.**

[Illustrative Mathematics](#)

8.EE Extending the Definitions of Exponents, Variation 1

Alignment 1: 8.EE.A.1

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

a. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study				0	1	2	3	4
Population (thousands)				2				

b. If you know the size of the population at a certain time, how do you find the population one hour later?

c. Marco said he thought that they could use the equation $P = 2t + 2$ to find the population at time t. Seth said he thought that they could use the equation $P = 2 \cdot 2^t$. Decide whether either of these equations produces the correct populations for $t = 1, 2, 3, 4$.

d. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study? What about 3 hours before?

e. If you know the size of the population at a certain time, how do you find the population one hour *earlier*?

f. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

g. Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

h. Use the context to explain why it makes sense that $2^{-n} = (\frac{1}{2})^n = \frac{1}{2^n}$. That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by $\frac{1}{2}$.

Commentary:
This is an instructional task meant to generate a conversation around the meaning of negative integer exponents. While it may be unfamiliar to some students, it is good for them to learn the convention that negative time is simply any time before $t = 0$.

Students will struggle to put their explanation for part (h) together. A teacher might want to have the students do parts (a) - (g) as a precursor to providing an explanation like the one given in the solution for part (h).

Solution: Solutions

a. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study	0	1	2	3	4
Population (thousands)	2	4	8	16	32

b. You multiply it by 2, since it doubled.

c. The values predicted by Seth's equation agree exactly with those in the table above; Seth's equation works because it predicts a doubling of the population every hour. Marco's doesn't because it doesn't double the new population you have – instead it is doubling the time. Marco's equation predicts a linear growth of only two thousand bacteria per hour.

d. Since the population is multiplied by 2 every hour we would have to divide by 2 (which is the same as multiplying by $\frac{1}{2}$) to work backwards. The population 1 hour before the study started would be

$$\frac{1}{2} \cdot 2 = 1 \text{ thousand},$$

and the population 3 hours before the study started would be

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 2 = 0.25 \text{ thousand} = 250.$$

e. Since the population is multiplied by 2 every hour we would have to divide by 2 (or multiply by $\frac{1}{2}$) to work backwards.

f. Time before the study started would be negative time; for example one hour before the study began was $t = -1$.

Hours into study	-3	-2	-1	0	1	2	3	4
Population (thousands)	$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$	$\frac{1}{2} \cdot 1 = 0.5$	1	2	4	8	16	32

g. Since one hour before the study started would be $t = -1$, we would simply plug this value into Seth's equation:

$$2 \cdot (2)^{-1} = 2 \cdot \left(\frac{1}{2}\right) = 1 \text{ thousand}.$$

Three hours before would be $t = -3$. Using the equation:

$$2 \cdot (2)^{-3} = \frac{2}{2^3} = 0.25 \text{ thousand},$$

giving us the same answers as we got through reasoning.

h. Since the bacteria double every hour, we multiply the population by two for every hour we go forward in time. So if we want to know what the population will be 8 hours after the experiment started, we need to multiply the population at the start ($t = 0$) by 2 eight times. This explains why we raise 2 to the number of hours that have passed to find the new population; repeatedly doubling the population means we repeatedly multiply the population at $t = 0$ by 2.

In this context, negative time corresponds to time *before* the experiment started. To figure out what the population was before the experiment started we have to "undouble" (or multiply by $\frac{1}{2}$) for every hour we have to go back in time. So if we want to know what the population was 8 hours before the experiment started, we need to multiply the population at the start ($t = 0$) by

$\frac{1}{2}$ eight times. The equation indicates that we should raise 2 to a power that corresponds to the number of hours we need to go back in time. For every hour we go back in time, we multiply by $\frac{1}{2}$. So it makes sense in this context that raising 2 to the -8 power (or any negative integer power) is the same thing as repeatedly multiplying $\frac{1}{2}$ 8 times (or the opposite of the power you raised 2 to). In other words, it makes sense in this context that

$$2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}.$$

8.EE Extending the Definitions of Exponents, Variation 1 is licensed by Illustrative Mathematics under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License

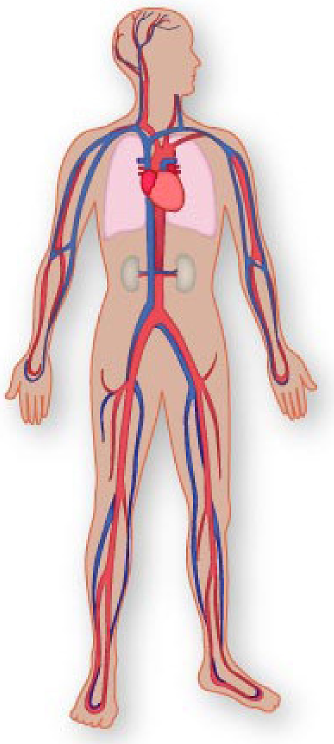
Exponents – Culminating Task Name________________________________

Blood in the Human Body Date_____________

Today you will be asked to solve several questions that require the use of the properties of exponents. Before you begin, please provide a convincing argument to show why the following rules are true.

1) Prove that $\frac{b^m}{b^n} = b^{m-n}$

2) Prove that $b^0 = 1$



Use the properties of exponents, as they apply, to solve the following problems.

3) There are 2.7×10^8 hemoglobin molecules in a single human red blood cell, and there are about 5.0×10^6 human red blood cells in one cubic mm of blood. How many hemoglobin molecules are in one cubic mm of blood? Show your work.

4) Irving claims he used a property of exponents to solve question 3. What property of exponents did Irving use? How do you know?

5) A red blood cell has a diameter of approximately 7.5×10^{-4} cm. Suppose one of the arteries in your body has a diameter of 4.56×10^{-2} cm. Irving says that 6.08×10^{2} red blood cells would fit across the artery. Do you agree or disagree with Irving? Why or why not?

6) If you donate blood regularly, the American Red Cross has had a policy of waiting 56 days between donations. They are currently re-assessing their policy. One pint of blood contains about 2.4×10^{12} red blood cells. Your body normally produces about 2×10^{6} red blood cells per second. What is your recommendation for waiting period between blood donations? Do you agree or disagree with the current policy of the American Red Cross?

8th Grade Exponents

"Blood in the Human Body"

Answer Rubric	**Points**	**Total Points**
1. • Sample Proof: Using the definition of an exponent, you can expand the numerator and the denominator and simplify the fraction by making ones, for example $\frac{b^5}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^3$. Or you can subtract the exponents, 5 – 2 = 3, to represent that the 2 b's in the denominator cancel (i.e. "make one") with 2 of the b's in the numerator.	1 - 2	**2**
2. • Sample Proof: Knowing the property for multiplying powers with the same base: $b^m \cdot b^n = b^{m+n}$, I could write an equation to solve for b^0 $b^5 \cdot b^0 = b^{5+0}$ $b^5 \cdot b^0 = b^5$ b^0 must equal one becaue anything multiplied by one is itself Therefore $b^0 = 1$ Or... • Sample Proof: Knowing the property for dividing powers with the same base: $\frac{b^m}{b^n} = b^{m-n}$, I could write an equation to solve for b^0 $\frac{b^5}{b^5} = b^{5-5} = b^0$. Anything divided by itself equals 1, so $\frac{b^5}{b^5} = 1$, so $1 = b^0$	1 – 2 1 – 2	**2**
3. • $(2.7 \times 10^8)(5.0 \times 10^6) = 1.35 \times 10^{15}$ hemoglobin molecules	1	**1**
4. • Irving used the additive property of exponents Or $b^m b^n = b^{m+n}$ • Student recognizes that in their work for problem 3 they multiplied two powers and therefore used the additive property of exponents.	1 1	**2**
5. • $(4.56 \times 10^{-2})/(7.5 \times 10^{-4}) = 6.08 \times 10^1$ • Student disagrees with Irving. • Irving said that $(4.56 \times 10^{-2})/(7.5 \times 10^{-4}) = 6.08 \times 10^2$, so it looks like he divided correctly at first to get $.608 \times 10^2$, but then didn't correctly put it in scientific notation. The correct answer, 6.08×10^1, is actually 10 times less than Irving's answer.	1 1 1 – 2	**4**

6. • The human body can produce $(2 \times 10^6)(60)(60)(24) = 1.728 \times 10^{11}$ red blood cells per day It would take $(2.4 \times 10^{12})/(1.728 \times 10^{11}) = 14$ days to replenish the red blood cells lost for donating one pint of blood. • Students disagree with the current policy of waiting 56 days between donations. • Students recommend a waiting period of 14 days or more between donations	1 - 2 1 1	**4**
TOTAL		**15**